JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 10, No. 1, January–March 1996

Technical Comments

Comment on "Corrective Term in Wall Slip Equations for Knudsen Layer"

Roop N. Gupta*
NASA Langley Research Center, Hampton, Virginia 23681
and
Carl D. Scott†
NASA Johnson Space Flight Center, Houston, Texas 77058

IN a recent Technical Note,[1] Goniak and Duffa obtained an additional term to the surface temperature slip equation of Ref. 2. This term has been referred to as a corrective term in Ref. 1. However, the derivation of this additional term should be viewed from the degree of approximation involved and the validity[3] of continuum methods to treat transition regime. The so-called correction term is, in fact, a higher-order term and may not be significant in the part of transition regime where a continuum flow analysis can be used, i.e., for $Kn < 1$ and Knudsen layer thickness of order mean free path. In their analysis, Gupta et al.[2] considered only the random thermal velocity and not the mean flow (or drift) velocity (which was considered negligible across the Knudsen layer). This is consistent with the approaches of Shidlovskiy[4] and Patterson,[5] who neglected the convection (or slip) velocity at the edge of Knudsen layer in their velocity distribution functions. Shidlovskiy assumed a nondrifting Maxwellian distribution function, whereas Patterson used a nondrifting Maxwellian with Hermite polynomial perturbation terms. In Ref. 2, a nondrifting Chapman–Enskog velocity distribution function was assumed. The additional (or correction) term of Ref. 1 results from the inclusion of mean flow (or drift) velocity in the analysis. The temperature slip equation with this additional term is[1]

$$T_s = T_w - [(2-\theta)/\theta](\tfrac{1}{2}RT)^{1/2}\sqrt{\pi}(T_s/P_s)q_y + \underline{(1/4R)u_s^2}$$

or, in the nondimensional form

$$\bar{T}_s = \bar{T}_w - \mathcal{O}(\varepsilon^2) + \mathcal{O}(\varepsilon^4)$$

with

$$\varepsilon^2 = \mu_{\text{ref}}/\rho_\infty U_\infty r_n$$

as defined in Ref. 2. Obviously, the additional term (underlined above) is of higher order as compared to the second term on the right-hand side. In fact, one should retain other terms of order ε^2 [see Eq. (73b) of Ref. 2] before this term is considered important in the temperature slip equation. Further, u_s must be near 100 m/s to increase the slip temperature T_s by 10 K through this additional term.[1]

Thus, there is no correction involved in the temperature slip equation of Ref. 2 for a formulation of order ε^2. For flow conditions, where terms of order ε^4 become important, a continuum approach to analyze transition flow regime may be called in question.[3]

Received July 7, 1995; accepted for publication Sept. 18, 1995.

*Senior Aerospace Engineer, Aerothermodynamics Branch. Associate Fellow AIAA.
†Senior Scientist, Aeroscience Branch. Associate Fellow AIAA.

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Reply by the Author to R. N. Gupta and C. D. Scott

G. Duffa*
Commissariat à l'Energie Atomique, 33114 Le Barp, France

IN a Technical Note,[1] Goniak and Duffa have given an expression for the temperature slip in a Knudsen layer. This expression contains an additive term compared to the previous work of Grad[2] and Gupta et al.[3]:

$$T_s = T_w - \frac{2-\theta}{2\theta}\cdot\frac{\beta_s T_s \pi^{1/2} q_y}{P_s} + \frac{u_s^2}{4R} \tag{1}$$

This expression is based on the solution of a problem in which the Knudsen layer is supposed collision-free. This hypothesis implies constant half fluxes of conservative quantities: mass, momentum, and energy. This second item was given by Grad without physical justification.

It was demonstrated that theoretical inconsistencies or differences in previous works are resolved by the last term in Eq. (1). This term is proportional to $(\beta_w u_s)^2$, the squared value of the slip velocity u_s divided by the most probable value $1/\beta_w$ of the velocity of a Maxwellian distribution at wall. This term scales with $1/\rho_w^2$ and then seems a priori to be small.[4,5]

Received Sept. 6, 1995; accepted for publication Sept. 18, 1995.

*Senior Scientist, CEA/CESTA, BP2.